a good agreement between the measuring results and the simulations has been observed and the advantage of a very find discretization has been shown. In the future, this method can be a powerful tool to calculate losses and propagation constants for planar MMIC waveguides, especially at high frequencies with a drastically reduced amount of computation time.

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Three-Port Hybrid Power Dividers Terminated in Complex Frequency-Dependent Impedances

Stanislaw Rosloniec

***Abstract*—A new CAD algorithm for design of two- and four-section three-port hybrid power dividers terminated with complex frequency-dependent impedances is described. The dividers under consideration are composed of lumped element resistors and noncommensurate transmission line sections whose characteristic impedances take extreme, practically realizable, values. These values are assumed freely at the beginning of a design process. The validity of the presented design algorithm has been confirmed by numerical modeling and experimentation.**

I. Introduction

The hybrid power dividers (combiners) of Wilkinson type are widely used in various UHF and microwave devices intended to work at small and medium power levels. As a rule, those dividers are terminated with the same frequency-independent resistances, usually equal to 50 Ω. Typical examples of such divider designs are described in the commonly available literature (see [1]–[4] for instance). In contrast to that, [5] and [6] present algorithms for designing similar dividers whose ports are loaded by different but also constant resistances. In some applications, however, it may be advantageous to use the dividers terminated with different and frequency-dependent complex impedances. By way of example, the combiners of that type are especially desirable for broad-band equiphase array antennas. Unfortunately, the design and hardware implementation of broad-band microwave dividers terminated in complex impedances have not as yet been sufficiently investigated in the literature. Therefore, this paper is a contribution to solve that practically important problem. A new CAD algorithm for design of novel Wilkinson dividers is described. The proposed dividers are composed of noncommensurate transmission line sections and lumped element resistors. It should be pointed out that characteristic impedances of these line sections are limited on both sides, i.e., by impedances $Z_{0\min}$ and $Z_{0\max}$ assumed freely at the beginning of a design process. Due to that, they may be easily realized, for example, of microstrip line segments. The validity of the proposed design method has been confirmed by numerical modeling and experimentation.

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II. Design Procedure

The strip line topologies of two- and four-section power dividers considered here are shown in Figs. 1(a) and 2(a), respectively. These divider circuits have mirror-reflection symmetry (with respect to planes $x-x'$), so they can be analyzed by means of the even- and odd-mode excitations method [7], [3]. Consequently, we obtain two pairs of bisection two-port circuits whose electrical schemes are shown also in Figs. 1 and 2. According to [1]–[3], [5], the scattering parameters $[S(f)]$ of power dividers under analysis may be expressed in terms of the scattering parameters $[S^{++}(f)]$ and $[S^{+-}(f)]$ evaluated for the corresponding even- and odd-mode two-port circuits. The suitable relationships are

$$\begin{aligned}
S_{11}(f) &= S_{11}^{++}(f)\\
S_{12}(f) &= S_{21}(f) = S_{13}(f) = S_{31}(f) = S_{12}^{++}(f)/\sqrt{2}\\
S_{22}(f) &= S_{33}(f) = [S_{22}^{++}(f) + S_{22}^{+-}(f)]/2\\
S_{23}(f) &= S_{32}(f) = [S_{22}^{++}(f) - S_{22}^{+-}(f)]/2.
\end{aligned} \tag{1}$$

It is evident from Figs. 1(b) and 2(b) that even-mode two-ports serve as broad-band stepped transmission line matching circuits included between the complex admittances $Y_g(f)/2$ and $Y_l(f)$. In this paper, the method published in [8] has been chosen for the design of these distributed element matching circuits. As it results from the relationships in (1), the divider characteristic $S_{11}(f)$ is unequivocally determined by the scattering parameter $S_{11}^{++}(f)$, i.e., indirectly by admittances $Y_g(f)/2, Y_l(f)$ and electrical parameters of the matching circuit connecting them. In other words, the characteristic $S_{11}(f)$ is independent of the isolating resistors. This feature allows us to shape the isolation characteristic $I(f)[\mathrm{dB}] = 20\log[1/|S_{23}(f)|]$ between divider ports 2 and 3 without distortion of the input return loss characteristic $S_{11}(f)[\mathrm{dB}] = 20\log[|S_{11}(f)|]$ achieved earlier. Therefore, let us assume that characteristic impedances and electrical lengths of the line sections creating the divider are known [8]. Then, the next nontrivial problem of the design is to calculate the isolating resistors such that characteristics $I(f)[\mathrm{dB}]$ and $S_{22}(f)[\mathrm{dB}] = S_{33}(f)[\mathrm{dB}] = 20\log[|S_{22}(f)|]$ will be optimum [3]–[5], [9]. It has been found numerically that this optimization problem may be successfully solved by using the minimization method presented below. For clarification of further considerations, let us assume that a four-section divider similar to that shown in Fig. 2 is designed. Of course, the above assumption does not lessen the generality of the proposed design approach which is based upon the equation

$$|S_{23}(f)| = |[S_{22}^{++}(f) - S_{22}^{+-}(f)]|/2 \quad (2)$$

where

$$S_{22}^{++}(f) = \exp(j2\phi)[Y_1^*(f) - Y_d^{(e)}(f)]/[Y_1(f) + Y_d^{(e)}(f)]$$

$$S_{22}^{+-}(f) = \exp(j2\phi)[Y_1^*(f) - Y_d^{(o)}(f)]/[Y_1(f) + Y_d^{(o)}(f)].$$

The admittance $Y_1^*(f)$ (used in the above relationships) is complex conjugated with the terminating admittance $Y_1(f) = |Y_1(f)|\exp(j\phi)$. From (2) it follows that the scattering parameter $|S_{23}(f)|$ reaches its zero minima when admittances $Y_d^{(e)}(f) = G_d^{(e)}(f) + jB_d^{(e)}(f)$ and $Y_d^{(o)}(f) = G_d^{(o)}(f) + jB_d^{(o)}(f)$ are equal to each other (see Fig. 2). Thus, in order to minimize the maximum value of the function $|S_{23}(f)|$, we have to ensure a closeness of these admittances over the required frequency band $(f_1 \div f_2)$. That problem, formulated in mathematical terms, may be written as

$$\min\max \frac{[G_d^{(e)}(f) - G_d^{(o)}(f)]^2 + [B_d^{(e)}(f) - B_d^{(o)}(f)]^2}{[G_d^{(e)}(f)]^2 + [B_d^{(e)}(f)]^2}$$

$$G \qquad f_1 \le f \le f_2 \quad (3)$$

where $G = (1/R_1, 1/R_2, 1/R_3, 1/R_4)$ is a four-dimensional vector of the lumped element conductances (resistances) being sought. The above problem can be effectively solved using the two-stage optimization procedure that follows. The first step in the first stage of this procedure is the choice of such a value of conductance $G_1 = 1/R_1$ for which the admittances $Y_a^{(e)}(f) = G_a^{(e)}(f) + jB_a^{(e)}(f)$ and $Y_a^{(o)}(f) = G_a^{(o)}(f) + jB_a^{(o)}(f)$ will be as near as possible in a sense of the following criterion:

$$\min\max \frac{[G_a^{(e)}(f) - G_a^{(o)}(f)]^2 + [B_a^{(e)}(f) - B_a^{(o)}(f)]^2}{[G_a^{(e)}(f)]^2 + [B_a^{(e)}(f)]^2}$$

$$G_1 \qquad f_1 \le f \le f_2. \quad (4)$$

Next, when the first approximation $G_1^{(1)}$ of conductance G_1 is known, it is possible to evaluate the first approximation of the conductance $G_2 = 1/R_2$ in the same manner. Of course, in this case, the one-dimensional optimization problem, like problem (4), is

$$\min\max \frac{[G_b^{(e)}(f) - G_b^{(o)}(f)]^2 + [B_b^{(e)}(f) - B_b^{(o)}(f)]^2}{[G_b^{(e)}(f)]^2 + [B_b^{(e)}(f)]^2}$$

$$G_2 \qquad f_1 \le f \le f_2 \quad (5)$$

where $G_b^{(e)}(f)$, $B_b^{(e)}(f)$, $G_b^{(o)}(f)$ and $B_b^{(o)}(f)$ are real and imaginary parts of admittances $Y_b^{(e)}(f)$) and $Y_b^{(o)}(f)$, respectively. It stands to reason that repeating the above routine for isolating resistors placed at planes $c - c'$ and $d - d'$ we obtain the third and fourth components of the vector $G^{(1)} = [G_1^{(1)}, G_2^{(1)}, G_3^{(1)}, G_4^{(1)}]$. The so-evaluated first approximation of the vector G serves as a starting point for the next (second) stage of the optimization procedure. The further minimization of the function $|S_{23}(f, G)|$ within a given frequency band can be continued by using the ϵ-steepest descent method [9]. For that, the following standard objective function is quite adequate [8]–[11]:

$$OF = \sum_{f_{ei} \subset Q} T^2[|S_{23}(f_{ei}, G)|]. \quad (6)$$

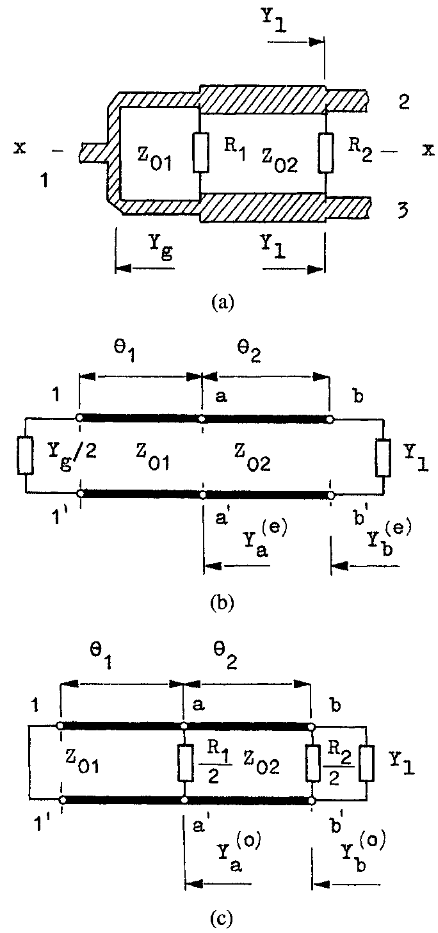


Fig. 1. Two-section power divider. (a) Overall divider structure. (b) Even-mode two-port circuit. (c) Odd-mode two-port circuit.

There, $T(x) = (1 + |x|)/(1 - |x|)$ is an auxiliary well-known transformation and Q is a set of discrete band frequencies f_{ei} for which the inequalities given below are satisfied:

$$0.97|S_{23}(f, G)|_{\max} \le |S_{23}(f_{ei}, G)| \le |S_{23}(f, G)|_{\max}. \quad (7)$$

For minimization of the objective function (6), regarding the conductances G_1, G_2, G_3, and G_4, the Davidon–Fletcher–Powell method has proved itself to be the most effective [10]–[12]. During the optimization process, all above-mentioned conductances have to be positive and naturally independent on frequency.

III. Results of the Theoretical and Experimental Investigation

In order to illustrate the design algorithm described in the previous paragraph, let us design a four-section hybrid power divider for the following input data: $Y_g(f) = (1/50)$ mho, $Y_l(f) = (1/50) + j(1/75)\tan(0.4f/f_0)$ mho, $f_0 = 1$ GHz, and a relative bandwidth $w = 2\Delta f/f_0 = 0.4$. The constructional constraints for that divider are determined by impedances $Z_{0\min} = 40\ \Omega$ and $Z_{0\max} = 80\ \Omega$. The characteristic impedances and electrical lengths of the divider sections evaluated for given data are: $Z_{01} = Z_{04} = 80\ \Omega$, $Z_{02} = Z_{03} = 40\ \Omega$, $\theta_1(f_0) = 1.8757$ rad, $\theta_2(f_0) = \theta_3(f_0) = 0.2646$ rad, and $\theta_4(f_0) = 0.6424$ rad [8]. The characteristic $VSWR_1(f) = [1 + |S_{11}(f)|]/[1 - |S_{11}(f)|]$ calculated for these parameters is illustrated in Fig. 3(a). Now, let us assume that five isolating resistors are located in planes $a - a'$, $b - b'$, $c - c'$, $d - d'$, and $e - e'$, characterized by the electrical lengths $\theta_a(f_0) = 0.6250$ rad, $\theta_b(f_0) = 0.6250$ rad,

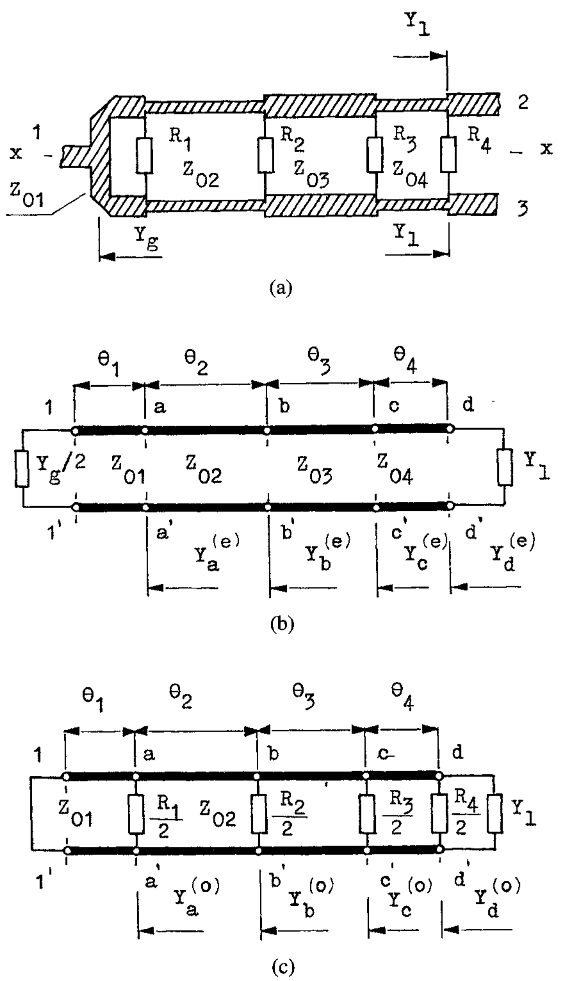


Fig. 2. Four-section power divider. (a) Overall divider structure. (b) Even-mode two-port circuit. (c) Odd-mode two-port circuit.

$\theta_c(f_0) = 0.6257$ rad, $\theta_d(f_0) = 0.5293$ rad, and $\theta_\epsilon(f_0) = 0.6424$ rad [see Fig. 4(a)]. The isolating resistors evaluated in the first stage of the design process are: $R_1 = 100.00\ \Omega$, $R_2 = 142.84\ \Omega$, $R_3 = 181.80\ \Omega$, $R_4 = 1998.90\ \Omega$, and $R_5 = 166.65\ \Omega$. The initial isolation characteristic $I_i(f)$[dB] calculated for the divider containing these resistors is shown in Fig. 3(c). During the second stage of the optimization, the resistors change considerably and take the following optimum values: $R_1 = 100.02\ \Omega$, $R_2 = 144.67\ \Omega$, $R_3 = 171.19\ \Omega$, $R_4 = 1190.23\ \Omega$, and $R_5 = 256.31\ \Omega$. Fig. 3(c) also presents the shape of the corresponding optimum isolation characteristic $I_{\rm opt}(f)$[dB]. The shape of the resulting output characteristics $VSWR_2(f) = VSWR_3(f) = [1 + |S_{22}(f)|]/[1 - |S_{22}(f)|]$ is given in Fig. 3(b). The divider designed above has been constructed of air triplate line segments characterized by a ground-to-ground spacing $b = 9.1$ mm and a center conductor thickness $t = 0.8$ mm [12], [13], Each terminating admittance $Y_l(f)$ is created by the constant resistance $Z_0 = 50\ \Omega$ connected in parallel with the 75 Ω open-circuited transmission line segment. The electrical length of that shunt line segment is equal to 0.4 rad at the center band frequency f_0 (see Fig. 4). The characteristics $VSWR_1(f) = [1 + |S_{11}(f)|]/[1 - |S_{11}(f)|]$, $VSWR_2(f) = VSWR_3(f) = [1 + |S_{22}(f)|]/[1 - |S_{22}(f)|]$ and $I(f)[\text{dB}] = 20 \log [|S_{23}(f)|]$ obtained experimentally are shown together with theoretical ones in Fig. 3. As it is seen, all experimental results confirm well the theoretical predictions. In the design example given above, the number of isolating resistors, and places where they are located, have been chosen freely. It is understood that these degrees of freedom make the optimization process (its first stage especially) more flexible and should be assumed rationally for every design problem. Generally, an increase of the number of isolating resistors yields better heat dissipation. Simultaneously, it allows improvement of the isolation characteristic $I(f)$[dB]. Another advantageous feature of the proposed algorithm is its generality. In fact, it is directly applicable to the design of two-section power dividers similar to that shown in Fig. 1.

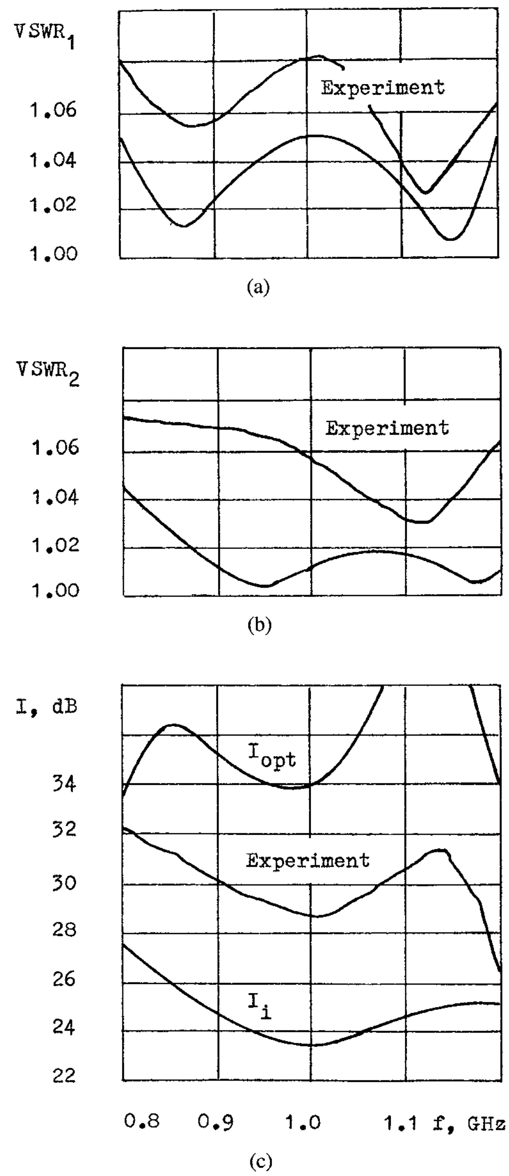
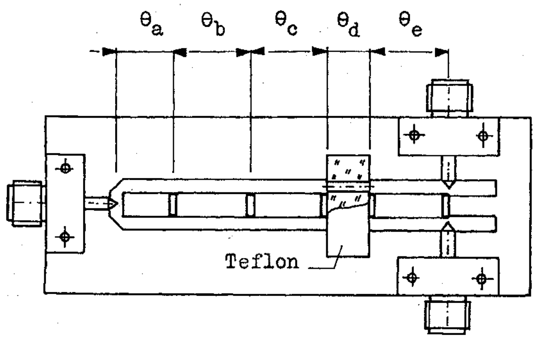


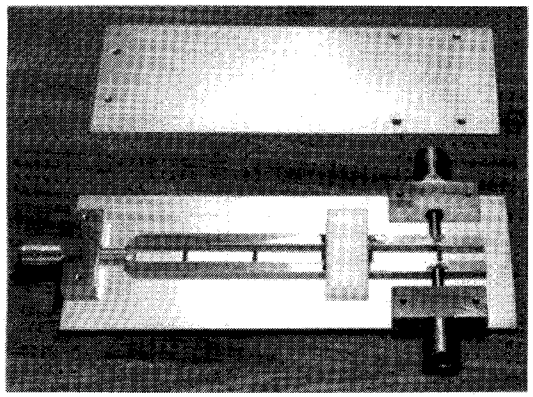
Fig. 3. The theoretical and experimental characteristics evaluated for the divider shown in Fig. 4.

IV. Conclusion

The new CAD algorithm for design of two- and four-section three-port hybrid power dividers terminated with different frequency-dependent complex impedances is described. The novel Wilkinson dividers are composed of the lumped element isolating resistors and noncommensurate TEM transmission line sections. The number of divider sections is evaluated on a basis of the return loss characteristic $S_{11}(f)$[dB]. If that characteristic calculated for the two-section divider does not satisfy the given requirements, then the more broad-



(a)



(b)

Fig. 4. Four-section power divider tested experimentally. (a) Air triplate line configuration. (b) Outside view.

band divider composed of four sections should be designed. The characteristic impedances of those line sections are limited by extreme impedances $Z_{0\min}$ and $Z_{0\max}$, which are assumed freely at the beginning of the design process [8]. Undoubtedly, that opportunity makes their realization easier. Moreover, in the analysis of the even- and odd-mode two-ports, only one type of step discontinuity has to be taken into account. It is justified by the fact that they are equivalent to each other. The compensation of these discontinuities can be achieved by means of the conventional techniques, for instance, as described in [12], [14], and [15]. Also, the parasitic shunt reactances of the isolating resistors can be easily included in the analysis of the odd-mode two-ports. In the first approximations, they are connected in parallel with the half-resistances being sought [see Figs. 1(c) and 2(c)]. The effective two-stage optimization procedure for calculating the isolating resistors is proposed. Presented numerical and experimental results confirm the validity of the proposed design algorithm and indicate that microwave dividers of this type may be adequate for some practical applications.

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Field Distributions in Supported Coplanar Lines Using Conformal Mapping Techniques

N. H. Zhu, E. Y. B. Pun, and P. S. Chung

***Abstract*— We use conformal mapping techniques to derive the analytical expressions for calculating the field distributions in supported coplanar lines. Our calculations agree well with the results obtained using the point matching method. Our method is an extension of the approximate technique proposed by Veyres and Hanna [1], and provides an accurate and fast calculation of the field distributions. This method can be extended to the analysis of other coplanar lines widely used in monolithic microwave integrated circuits (MMIC) and optical integrated circuits (OIC) applications.**

I. INTRODUCTION

Coplanar lines have been investigated extensively in monolithic microwave integrated circuits (MMIC's). The quasistatic analyzes of various coplanar lines are well studied using the conformal mapping techniques [1]–[8]. The efforts have been directed mainly toward deriving the approximate analytical formulas for quasistatic parameters, such as effective dielectric constant and characteristic impedance. The point matching method [9] can be used to calculate the field distributions. Unfortunately, its solutions display serious

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